Finite Difference Equations

Expressing **Equation 13** in fully implicit finite difference form (i.e., all Q values are at the t+Dt time step, or q=1:

Note: Henceforth all bold variables are the mean conduit values:

Equation 18:

$$Qt+\Delta t = Qt - g\cdot k\cdot \Delta t\cdot |V_t| Q_{t+\ \Delta t} / \mathbf{R}^{4/3} - \mathbf{V}\ (\Delta A/\Delta t)\cdot \Delta t + Q_{t+\ \Delta t}\cdot [1/A_{up} - 1/A_{dn}]/L - g\cdot \mathbf{A}\cdot [(H_{dn} - H_{up})/L]\cdot \Delta t$$

$$Qt+Dt = Qt - g\cdot k\cdot Dt\cdot |V_t| Q_{t+\ Dt} / \mathbf{R}^{4/3} - \mathbf{V}\ (DA/Dt)\cdot Dt + Q_{t+\ Dt}\cdot [1/A_{up} - 1/A_{dn}]/L - g\cdot \mathbf{A}\cdot [(H_{dn} - H_{up})/L]\cdot Dt \quad (18)$$

Solving equation 18 for $Q_{t+\ \Delta t}$ 1gives the final finite difference form of the fully implicit dynamic flow equation (excluding terms for Sc and Se),

Equation 19:

$$Q_{t+\Delta t} = \frac{\left[Q_t + \mathbf{V}\cdot \Delta A/\Delta T\cdot \Delta t - g\cdot \mathbf{A}\cdot (H_{dn} - H_{up})/L\cdot \Delta t\right]}{\frac{\left[g\cdot k\cdot |\mathbf{V}|\cdot \Delta t\right.}{\mathbf{R}^{4/3}} - \frac{Q_t\cdot (1/A_{up} - 1/A_{dn})\cdot \Delta t\ ]}{L}}$$

$(\Delta A/\Delta T)$ is the average area time derivative from time step n.