Special Finite Difference Approximations

A flow retardation parameter, f, is used for conduits that have slight or flat slopes. The definition of a slight slope as used in the model is conduits with slopes ≤ 1/10,000. The default value of f (currently 0.05) is used to moderate the solution of the conduit flow.

The above parameter is used as follows in EXTRAN:

Equation 53: $$\frac{Q_{n+1} - Q_n}{\Delta t} + \frac{\phi \cdot Q_{n+1}}{\Delta t} + LHS = RHS$$

where:

LHS = dynamic wave equation variables at the new time step or iteration, and

RHS = dynamic wave equation variables at the old time step or iteration.

Generally, the solution of the flow at the new time step (Q_{n+1}) is solved as follows:

Equation 54: $$Q_{n+1} = \frac{Q_n + RHS \cdot \Delta t}{1 + f + LHS \cdot \Delta t}$$

This solution is further moderated by using under-relaxation on the new values of RHS and LHS. This control makes the solution smoother and eliminates the growth of potential instabilities.

Equation 55: $LHS_{n+1} = (1\text{-}w) \cdot LHS_n + w \cdot LHS_{n+1/2}$

Equation 56: $RHS_{n+1} = (1\text{-}w) \cdot RHS_n + w \cdot RHS_{n+1/2}$

The combined pressure-bed slope term in the dynamic wave equation is solved at the new and old time step by factoring out a Q from the A in $g \cdot A \cdot \partial H / \partial x$ as follows:

Equation 57: $$Q_{n+1} \cdot \frac{A_{n+1/2}}{Q_{n+1/2}}$$

EXTRAN uses an iterative solution to solve the gradually varied unsteady flow (St. Venant) equations. The convergence criteria for this method is related to the maximum number of iterations and the relative accuracy of nodal and conduit computations as specified in the simulation tolerances dialog.

The weighting coefficient for the non-linear momentum equation is calculated based on the conduit Froude number. Furthermore, when both ends of the conduit are surcharged the non-linear acceleration term is neglected by the program.

The finite difference approximation for the $V \cdot \partial A / \partial t$ term is calculated as follows in EXTRAN:

First Iteration:

Equation 58: $$(V_{up} + V_{dn})^n \cdot \frac{(A_n - A_{n-1}) \cdot \Delta t}{2 \cdot (\Delta t + \Delta t_1)}$$

Subsequent Iterations:

Equation 59: $$(V_{up} + V_{dn})^n \cdot \frac{(A_{n+1/2} - A_n) \cdot \Delta t}{2 \cdot (.t + t_1)}$$

The model does not allow a flow reversal in one time step. The intervening flow is 1/1000 the previous conduit flow. This prevents oscillations in the conduit from arising from alternating positive and negative flows. This reversal check is performed on all conduits as well as orifices, pumps and weirs. The minimum flow in any conduit or diversion is set at 1.0e-10 (cms or cfs) to prevent underflow and overflow errors using MICROSOFT FORTRAN.

Control over the flow direction is also possible with internal flap gates in conduits that only allow flow either from the upstream to the downstream node or from the downstream to the upstream node.

The vertical differentiation of conduit roughness as defined by the special conduit factor dialog is a quadratic equation ranging from the peak at zero depth to normal roughness at the transition depth.

Initial transients in the model are dampened by increasing the roughness coefficient of each conduit by a factor of ten. This optional technique decreases oscillations at the beginning of a simulation. If used, the transition to normal roughness takes 9 large time steps (Δt).

The combined momentum-continuity equation uses the difference of the average cross-section area (A) at the n-1 and n+1 time steps with the denominator being the larger Δt plus the sum of the smaller time steps (Δt). The numerator uses the average velocity at the current iteration.