Fundamentals of Hydraulic Transient

Hydraulic transient describes the disturbances in a fluid caused during a change, typically from one steady-state equilibrium condition to another. The principle components of the disturbances are pressure changes caused by the propagation of pressure waves throughout the distribution system. These pressure waves travel with the velocity of sound (acoustic or sonic speed) and continue to propagate creating transient pressure and flow conditions until they are dissipated down (stabilized) to the level of the new steady-state by the action of some form of damping or friction. Normally, only if the flow is regulated extremely slowly then it is possible to undertake a smooth transition from one steady-state to another without large fluctuations in pressure or flow.

Disturbances in water distribution systems originate from changes or actions that affect hydraulic devices or boundary conditions including:

1. pump startup or shutdown
2. valve opening or closing
3. changes in boundary pressures (reservoir level, pressure tank, etc.)
4. rapid changes in demand conditions (e.g., hydrant flushing)
5. changes in transmission conditions (e.g., pipe break)
6. pipe filling or draining

The fundamentals of hydraulic transient in water distribution systems are developed on the basis of the basic conservational relationships of physics or fluid mechanics and the way typical hydraulic devices interact with the wave conditions in the pipes. They can be fully described by three physical relations:

- Newton’s Second Law (Equation of Motion),
- Conservation of Mass (Kinematics Relation),
- The Equation of State (Compressibility considerations which lead to a Wave-Speed Relation).

If $x$ is the distance along the pipe centerline, $t$ is the time, and partial derivatives are represented as subscripts, then the governing equations for transient flow can be written as:

**Continuity**

$$ H_t + \frac{c^2}{gA} Q_x = 0 $$

(1)

**Momentum (dynamic)**

$$ H_x + \frac{1}{gA} Q_t - f(Q) = 0 $$

(2a)

Neglecting friction in the above Equation and coupling with the Continuity Equation give the Joukowsky Relation:

$$ \Delta H = \frac{c}{gA} \Delta Q $$

(2b)

**Equation of State**

$$ c = \gamma \sqrt{\frac{K}{\rho}} $$

(3)

where $H$ is the pressure head (pressure/density); $Q$ is the volumetric flow rate; $c$ is the sonic wave speed in the pipe; $A$ is the cross sectional area; $g$ is the gravitational acceleration; $f(Q)$ is a pipe resistance term which is a function of flow rate; $\gamma$ is the ratio of the specific heats for the fluid (typically assumed equal to 1); $K$ is the bulk modulus of the fluid; and $\rho$ is the fluid density.

The sonic speed $c$ is influenced by the elasticity of the pipe wall. For a pipe system with some degree of axial restraint, a good approximation for the wave propagation speed is obtained using:

$$ c = \sqrt{\frac{E_f}{\rho} \left(1 + \frac{K_r E_f D}{E_c t} \right)} $$

(4)
where $E_f$ and $E_c$ are the elastic modulus of the fluid and conduit respectively; $D$ is the pipe diameter; $t$ is the pipe thickness; and $K_R$ is the coefficient of restraint for longitudinal pipe movement. The constant $K_R$ takes into account the type of support provided for the pipeline.

Typically, three cases are recognized with $K_R$ defined for each as follows ($\mu$ is the Poisson’s ratio for the pipe material):

Case a: The pipeline is anchored at the upstream end only.

$$K_R = 1 - \frac{\mu}{2}$$

Case b: The pipeline is anchored against longitudinal movement.

$$K_R = 1 - \mu^2$$

Case c: The pipeline has expansion joints throughout.

$$K_R = 1 - \mu^2$$

Table 1 gives physical properties of common pipe materials.

### Table 1 – Physical Properties of Common Pipe Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus E (GPa)</th>
<th>Poisson’s Ratio $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos Cement</td>
<td>23 - 24</td>
<td>-</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>80 - 170</td>
<td>0.25 - 0.27</td>
</tr>
<tr>
<td>Concrete</td>
<td>14 - 30</td>
<td>0.1 - 0.15</td>
</tr>
<tr>
<td>Reinforced Concrete</td>
<td>30 - 60</td>
<td>-</td>
</tr>
<tr>
<td>Ductile Iron</td>
<td>172</td>
<td>0.3</td>
</tr>
<tr>
<td>PVC</td>
<td>2.4 - 3.5</td>
<td>0.46</td>
</tr>
<tr>
<td>Steel</td>
<td>200 - 207</td>
<td>0.30</td>
</tr>
</tbody>
</table>

A plot of wave propagation speeds for water flowing in a circular pipe subjected to a representative pipeline restraint for a variety of materials is shown in Figure 1. For the majority of hydraulic analyses involving transients, the wave speed can be considered to be constant.
The primary Equations (1) and (2) governing unsteady flow in pipes are nonlinear hyperbolic partial differential equations for which no analytical solution exists except for very simple applications that neglect or greatly simplify the boundary conditions and the pipe resistance term. When pipe junctions, pumps, surge tanks, air vessels and other hydraulic components are included, the basic equations are further complicated. As a result, numerical solutions are used to integrate or solve the transient flow equations.

The primary Equations (1) and (2) governing unsteady flow in pipes are nonlinear hyperbolic partial differential equations for which no analytical solution exists except for very simple applications that neglect or greatly simplify the boundary conditions and the pipe resistance term. When pipe junctions, pumps, surge tanks, air vessels and other hydraulic components are included, the basic equations are further complicated. As a result, numerical solutions are used to integrate or solve the transient flow equations.